[MODEL QUESTION PAPER]
B-Tech
FIRST SEMESTER EXAMINATION-2008-09
MATHEMATICS-I

Time – 3 hours **Maximum marks :100**

Note : The Question paper contains Three sections, Section A, Section B & Section C with the weightage of 20, 30 & 50 marks respectively. Follow the instruction as given in each sections.

SECTION – A

This question contains 10 Questions of multiple choice/ Fill in the blanks/ True, False/ Matching correct answer type questions. attempt all parts of this section. [10x2=20]

Q1.(a) The characteristics values of the matrix $\begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix}$ are given as ____________________.

(b) If x = r Cos θ and y= r sin θ then the value of $\frac{\partial (xy)}{\partial(r\theta)}$ is ________________

(c) If $y = \sin^3 x$ then the N_{th} derivative (y_n) is ________________

(d) The value of the constant 'b' for a solenoidol vector $(bx + 4y^2z)\,\hat{i} + (x^3 \sin z - 3y)\hat{j} - (e^x + 4\cos x^2y)\,\hat{k}$ is __________________.

Pick the correct answer of the choices given :

(e) The matrix

$$\begin{pmatrix} i & 0 & 0 \\ 0 & 0 & i \\ 0 & i & 0 \end{pmatrix}$$ is

(i) Hermition Matrix only

(ii) Skew Hermition Matrix only

(iii) Hermition & Unitary both

(iv) Skew Hermition & Unitary both

(f) The curve represented by the equation $x^5 + y^5 = 5\,a^2x^2y$ is

(i) Symmetric about x – axis

(ii) Symmetric about y – axis

(iii) Symmetric about both x & y axis

(iv) None of these

Match the items on the right hand side with those on left hand side

(g)(i) $\Gamma 1$ (p) $\pi \backslash \sin n\pi$

(ii) $\Gamma n+1$ (q) $2 \int_0^\infty e^{-x^2} x^{2n-1}$

(iii) $\Gamma n \; \Gamma 1-n$ if $0 < n < 1$ (r) 1

(iv) Γn (t) $\lfloor n$

(h) (i) For scalar φ, $\nabla\phi$ is (p) $\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2}$

(ii) For Solenoidal vector $\overline{\phi}$, $\overline{\nabla}.\varphi$ is (q) 0

(iii) For Vector $\overline{\phi}$ then $(\nabla \times \phi)$ is (r) irrotational

(iv) For scalar φ, div grad ϕ is (s) $\hat{i}\frac{\partial \varphi}{\partial x} + \hat{j}\frac{\partial \varphi}{\partial y} + \hat{k}\frac{\partial \varphi}{\partial z}$

<u>Indicate True or False for the following Statements</u>:

(I) (i) if u , v are function of r, s are themselves functions of x, y then $\frac{\partial(uv)}{\partial(xy)} = \frac{\partial(uu)}{\partial(rs)} \times \frac{\partial(xy)}{\partial(rs)}$ **True / False.**

(ii) If z = f(xy) then the total differential of z, denoted by dz, is given as

$$dz = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy$$

True / False.

(J) (i) The function f(xy) is said to have maximum at thepoint (a,b) if f(ab) < f(a +h, b +k) for small positive or negative value of h & k. **True / False**.

(ii) If f(xyz) is a homogeneous function of Three independent variables (x,y,z) if order n , then

$$x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} + z \frac{\partial f}{\partial z} = n(n+1)\ f(xyz)$$ **True/ False**

SECTION – B

Note : Attempt any Three questions. All questions carry equal marks : [10x3]

Q2.(a) Find the Inverse of the matrix employing the elementary transformation

$$\begin{pmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{pmatrix}$$

(b) If $y = \sin [\ \text{Log} (x^2 + 2x + 1\ ]$, then prove that
$(1 + x)^2 y_{n+2} + (2n+1)(1 + x) y_{n+1} + (n^2 + 4) y_n = 0$

(c) If $x = \sqrt{vw}$, $y = \sqrt{wu}$, $z = \sqrt{u\,v}$ and $u = r \sin\theta . \cos \varphi$, $v = r \sin \theta \sin \varphi$, $w = r \cos \theta$ then calculate the Jacobian $\frac{\partial (xyz)}{\partial (r\ \theta\ \varphi)}$

(d) Evaluate $\iiint xyz\ dxdy\ dz$ for all positive values of variables through out the ellipsoid.

(e) Evaluate $\oint \bar{f}\, d\bar{r}$ by stokes theorem where $\bar{f} = (x^2 + y^2)\ \hat{i} - 2xy\hat{j}$ and 'c' is the boundary of the rectangle $x = \pm a$, $y = 0$ and $y = b$

SECTION – C

Note : Attempt any Two parts from each question. All questions are compulsory. [10x5 = 50]

Q3.(a) If $u = \log (x^3 + y^3 + z^3 - 3xyz)$ show that

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \right)^2 u = - \frac{9}{(x + y + z)^2}$$

(b) Find the Taylor's Series expression of the function $e^x \cos y$ at (0,0) upto five terms

(c) Trace the curve $y^2(2a - x) = x^3$

Q4.(a) If the radius of sphere is measured as 5 cm with a possible error of 0.2 cm. Find approximately the greatest possible error and percentage error in the compound value of the volume.

(b) Fins the point on the plane $ax + by + cz = p$ at which the function $f = (x^2 + y^2 + z^2)$ has a maximum value and hence the maximum.

(c) Find the dimensions of a rectangular closed box of maximum capacity whose surface is given.

Q5.(a) Verify the Caylay's Hamilton theorem for the matrix

$$\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$$

(b) Find the matrix P which diagonalizes the matrix A

$$A = \begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix}$$

(c) For different values of 'K' , discuss the nature of solutions of following equations –

$$x + 2y - z = 0$$
$$3x+(k+7)y - 3z = 0$$
$$2x + 4y + (k - 3) z=0$$

Q6.(a) Solve by changing the order of Integration

$$\int_0^{\infty} \int_{\sqrt{a2 - y2}}^{y+a} f(xy)\, dx\, dy$$

(b) Find the mass of an octant of the ellipsoid. $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, the density at any point being $\rho = k\, xyz$.

(c) Determine the area bounded by the curves $xy = 2$, $4y = x^2$ and $y = 4$

Q7.(a) If $\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |r|$

(i) $\text{div} \frac{\bar{r}}{|\bar{r}|^3} = 0$

(ii) $\text{div} (\text{grad}\, r^n) = n (n+1) r^{n-2}$

(b) Show that $\nabla \times (\nabla \times A) = \nabla (\nabla . A) - \nabla^2 A$

(c) Evaluate $\iint \bar{F}.\hat{N}\, ds$, where $\bar{F} = (4x\,\hat{i} - 2y^2\,\hat{j} + z^2\,\hat{k}$ and 's' is the region bounded by

$y^2 = 4x$, $x = 1$, $z=0$, $z = 3$